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Stocks, Bonds, and Long-Run Consumption Risks

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Abstract

I evaluate whether the so-called long-run risk framework can jointly explain key features of both equity and bond markets as well as the interaction between asset prices and the macroeconomy. I find that shocks to expected consumption growth and time-varying macroeconomic volatility can account for the level of risk premia and its variation over time in both markets. The results suggest a common set of macroeconomic risk factors operating in equity and bond markets. I estimate the model using a simulation estimator that accounts for time aggregation of consumption growth and utilizes a rich set of moment conditions.

I. Introduction

The challenge of understanding the dynamics of equity and bond markets has generated a large number of representative-agent models. However, it is common in the literature to treat the 2 markets in isolation rather than modeling them jointly. This is despite the fact that the representative agent's stochastic discount factor should be able to price stocks and bonds simultaneously, especially given today's integrated financial markets. In this paper, I evaluate whether the so-called long-run risk framework can jointly explain key features of both equity and bond markets as well as the relation between asset prices and the macroeconomy.

I find that persistent shocks to expected consumption growth together with a negative correlation between inflation and consumption growth are able to explain the average level of risk premia found in both equity and bond markets, while time variation in macroeconomic volatility can account for evidence of predictability across both markets. This suggests a common set of macroeconomic risk factors operating in equity and bond markets. The model does well in reproducing features of data such as the equity premium, the upward-sloping nominal yield curve,

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and the ability of price-dividend ratios and nominal yield spreads to predict future asset returns, economic growth, and macroeconomic volatility.

I estimate the model using simulated method of moments (SMM) and quarterly U.S. data for the period 1952:2–2007:4. The use of SMM alleviates problems stemming from time aggregation of consumption growth and stands in contrast to calibration, which is commonly used in the long-run risk literature.[1] Two exceptions are Bansal, Kiku, and Yaron (2007) and Bansal, Gallant, and Tauchen (2007), who estimate the long-run risk model using simulation estimators. However, these papers consider only a limited set of moment conditions and focus exclusively on equity markets. In contrast, I estimate the full model using a rich set of model-based restrictions covering macro variables, equity markets, and bond markets.

I use 2 different measures of inflation to estimate the model: the price index that corresponds to the National Income and Product Accounts (NIPA) consumption data and the Consumer Price Index (CPI). The model is rejected using the 1st inflation series, while the model cannot be rejected using the CPI. The difference mainly stems from the higher volatility of the CPI, which helps the model to better match the volatility of nominal yields. Considering that χ^2 tests have a tendency to overreject, the model must be said to offer a reasonable fit to the data.

Matching the predictive power of the yield spread and the price-dividend ratio imposes identifying restrictions on the elasticity of intertemporal substitution (EIS) and is therefore used as moment conditions in the estimation. This helps identify the value of the EIS and contributes to the literature on whether the EIS is close to 0 (e.g., Hall (1988), Campbell (1999)) or is above 1 (e.g., Bansal, Kiku, and Yaron (2007), Vissing-Jorgensen and Attanasio (2003)). I estimate the EIS to 2.51 with a standard error of 0.74 and the risk-aversion coefficient to 6.78 with a standard error of 1.61. I show that setting the EIS close to 0 generates counterfactual implications for predictability.

Real bonds in the model act as a hedge against bad times, as they perform well in periods of low consumption growth and high macroeconomic uncertainty. This produces negative risk premia on real bonds and a downward-sloping real yield curve. This is supported by empirical evidence from U.K. index-linked bonds (e.g., Evans (1998), Piazzesi and Schneider (2006)).[2] In contrast, nominal bonds are risky assets, as U.S. inflation is estimated to be countercyclical. High inflation in periods of low growth implies procyclical nominal bond returns, which generates a positive risk premium on nominal bonds that increases with the maturity of the bond. This makes the nominal yield curve slope upward on average, allowing the model to match the data.

Time-varying volatility of consumption growth gives rise to time-varying equity and bond risk premia in the model. An increase in macroeconomic uncertainty

[1]The temporal aggregation of reported consumption data has been shown to have potentially important effects when estimating asset pricing models (e.g., Heaton (1993), (1995), and Bansal, Kiku, and Yaron (2007)).

[2]Data for U.S. index-linked bonds only date back to 1997 and indicate a positively sloped real yield curve on average. This evidence should be interpreted with caution, as the time series is rather short and the market was illiquid at the inception of trading.

raises expected returns on equity and nominal bonds while steepening the nominal yield curve. This produces a violation of the expectations hypothesis. Running the Fama and Bliss (1987) regressions of bond excess returns onto the forward-spot spread in the model yields positive regression coefficients, indicating predictable bond returns. The model also captures the tent-shaped coefficients found in Cochrane and Piazzesi (2005). However, the explanatory power of the model regressions are smaller than what is observed in the data.

The nominal yield curve predicts future economic growth and excess stock returns positively in the model, which is consistent with the data. The ability of the nominal term spread to forecast future economic activity has been documented by several studies (e.g., Stock and Watson (1989), Estrella and Hardouvelis (1991), and Ang, Piazzesi, and Wei (2006)). The model explains this finding through the countercyclical nature of U.S. inflation together with a high value of the EIS.

The long-run risk framework of Bansal and Yaron (2004) contains 3 main features. First, the representative agent has Epstein and Zin (1989) recursive preferences, which allows the risk-aversion coefficient to be separated from the EIS.[3] Second, expected consumption growth is subject to highly persistent shocks that represent long-run risks of consumption. Third, the variance of consumption growth varies over time and produces a time-varying risk premium on assets. Consumption growth being non-independent and identically distributed (non-IID) is an important feature of the model. In order to price nominal bonds, I introduce an inflation process that allows for a correlation between the nominal and real sides of the economy.

This paper relates to the large literature on pricing stocks and bonds in equilibrium. In equity markets, Campbell and Cochrane (1999) present a habit-formation model with independent and identically distributed (IID) consumption growth that successfully matches asset prices.[4] Bansal and Yaron (2004) suggest a model with recursive preferences and non-IID consumption growth that also matches key features of equity markets. Although Brandt and Wang (2003), Wachter (2006), and Buraschi and Jiltsov (2007) provide evidence that variants of habit models are able to match observed interest rates while replicating deviations from the expectations hypothesis, the literature has been silent on whether the long-run risk model is able to jointly match key moments in bond and equity markets.[5] Piazzesi and Schneider (2006) explore term-structure implications in a related model, but risk premiums are constant and equity is not considered. Gallmeyer, Hollifield, Palomino, and Zin (2007) include a Taylor rule in a long-run risk setup and demonstrate that it produces realistic moments for interest rates, but risk premiums are constant and they do not consider equity. Eraker (2008) demonstrates that a continuous-time version of Bansal and Yaron (2004)

[3]Other papers that make use of recursive preferences in asset pricing include Campbell (1993), (1996), (1999), Duffie, Schroder, and Skiadas (1997), and Restoy and Weil (2011).

[4]An incomplete list of early contributions for equity markets is Sundaresan (1989), Abel (1990), (1999), Constantinides (1990), and Constantinides and Duffie (1996).

[5]Some notable contributions for bond markets are Cox, Ingersoll, and Ross (1985), Dunn and Singleton (1986), Campbell (1986), Backus, Gregory, and Zin (1989), and Donaldson, Johnsen, and Mehra (1990).

can match observed yield curve moments, but he does not consider time-varying risk premiums. In contrast to these papers, I show that the long-run risk framework is able to jointly explain properties of equity and bond markets, including evidence of predictability found in both markets.

This paper is contemporaneous with Bansal and Shaliastovich (2009), who provide evidence that the long-run risk model is able to generate rejections of the expectations hypothesis and explain the forward-premium puzzle. My paper differs from theirs in several aspects. For example, I estimate the model formally using a rich set of moment conditions as opposed to calibrating the model. I also evaluate the model's ability to simultaneously match evidence of predictability found in both equity and bond markets as well as cross moments between macro variables and asset prices. These features of the data are important to capture for any model that prices stocks and bonds jointly.

II. The Model

This section provides the macro dynamics, the preferences of the representative agent, and the solutions for bond prices. For simplicity, I choose to model the real side of the economy as in Bansal and Yaron (2000), (2004).

A. Dynamics

The real economy is subject to the following main processes:

$$
\begin{aligned}
&(1) & g_{t+1} &= \mu + x_t + \sigma_t \eta_{t+1}, \\
&(2) & x_{t+1} &= \rho x_t + \varphi_e \sigma_t \varepsilon_{t+1}, \\
&(3) & g_{d,t+1} &= \mu_d + \phi x_t + \varphi_d \sigma_t u_{t+1}, \\
&(4) & \sigma^2_{t+1} &= \sigma^2 + \nu_1 \left(\sigma^2_t - \sigma^2\right) + \sigma_w w_{t+1}, \\
& & \eta_{t+1}, \varepsilon_{t+1}, w_{t+1}, u_{t+1} &\sim \text{IID-Normal}\,(0, 1)\,.
\end{aligned}
$$

The log growth rate of consumption is denoted g_{t+1} and is determined by the unconditional mean μ, a persistent component x_t, and a shock η_{t+1}, which represents short-run risks to consumption. The persistent part x_t serves as a state variable and is affected by shocks ε_{t+1}, whose persistence is governed by ρ. These shocks affect the conditional mean of consumption growth and represent long-run risks of consumption.[6] The 2nd state variable is the conditional variance of consumption growth, σ^2_{t+1}. It is also subject to shocks w_{t+1}, which produce time-varying macroeconomic uncertainty. This is referred to as volatility risk. Consumption growth being non-IID is a crucial feature of the model. Dividend growth, $g_{d,t+1}$, is modeled as a function of expected consumption growth subject to a leverage parameter ϕ.

[6]Consider the revision of the conditional mean of consumption growth for a horizon of n periods, $\mathrm{E}_t(g_{t+n}) - \mathrm{E}_{t-1}(\mathrm{E}_t(g_{t+n})) = \rho^{n-1}\varphi_e\sigma_{t-1}\varepsilon_t$. This revision is 0 when φ_e equals 0.

I introduce the following inflation process in order to price nominal bonds:

$$\begin{aligned}
(5)\qquad \pi_{t+1} &= \mu_\pi + x_t^\pi + \delta_1 \sigma_t \eta_{t+1}^\pi, \\
(6)\qquad x_{t+1}^\pi &= \rho_\pi x_t^\pi + \delta_2 \sigma_t \varepsilon_{t+1} + \delta_3 \sigma_t \varepsilon_{t+1}^\pi, \\
\eta_{t+1}^\pi, \varepsilon_{t+1}^\pi &\sim \text{IID-Normal}\,(0,1)\,.
\end{aligned}$$

The log inflation rate is denoted π_{t+1} and is governed by its unconditional mean μ_π, expected inflation x_t^π, and a shock term $\delta_1 \sigma_t \eta_{t+1}^\pi$. Expected inflation is modeled as an autoregressive process that is affected by shocks to expected consumption growth through δ_2. Shocks to both realized and expected inflation are heteroskedastic. All shocks in the economy, real and nominal, are uncorrelated.

For parsimonious reasons, the volatility of inflation and consumption growth are governed by the same process.[7] This seems to be a reasonable restriction, considering that uncertainty measures of future inflation and economic growth are highly and positively correlated in the data.[8] The notion of heteroskedasticity in inflation is a well-established empirical fact; early contributions include Engle (1982) and Bollerslev (1986). The specification of inflation allows for a correlation between inflation and the real economy and is similar to the dynamics used in, for example, Campbell and Viceira (2001) and Piazzesi and Schneider (2006). However, in contrast to them, I allow for heteroskedasticity. This way of modeling inflation is a reduced-form approach for capturing the correlation between economic growth and inflation. A negative (positive) δ_2 leads to a negative (positive) correlation between growth and inflation in the model. The model is silent on what the actual mechanisms behind the inflation-growth relation are. One possible interpretation is that the sign of δ_2 reflects whether the economy has been subject predominantly to demand or supply shocks, since the former tend to be associated with procyclical inflation while the latter are often associated with countercyclical inflation. An alternative approach would be to endogenize inflation by allowing monetary policy to play a role through a particular interest-rate rule as in Gallmeyer et al. (2007).

B. Investor Preferences

The representative agent in the economy has Epstein and Zin (1989) preferences:

$$(7)\qquad U_t = \left\{ (1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left(\mathrm{E}_t \left[U_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}},$$

where $\theta = (1-\gamma)/(1-(1/\psi))$, $\gamma \geq 0$ denotes the risk-aversion coefficient and $\psi \geq 0$ the EIS. The discount factor is denoted δ. This specification allows

[7]Introducing a separate volatility process for inflation would add 1 more state variable but is straightforward. Derivations are available from the author.

[8]More specifically, uncertainty measured as the standard deviation of individual forecasters taken from the Survey of Professional Forecasters has a correlation of 0.68 for 1-quarter-ahead forecasts and 0.80 for 1-year-head forecasts. Economic growth is measured by real gross domestic product (GDP) growth and inflation by the GDP deflator. The time period is 1968Q4–2007Q4.

time preferences to be separated from risk preferences but nests the case of time-separable expected utility, in which case $\gamma = 1/\psi$ and $\theta = 1$. The agent prefers early (late) resolution of risk when the risk aversion is larger (smaller) than the reciprocal of the EIS. A preference for early resolution and an EIS above 1 imply that $\theta < 1$.

The agent is subject to the budget constraint $W_{t+1} = R_{a,t+1}(W_t - C_t)$, where the agent's total wealth is denoted W_t, $W_t - C_t$, the amount of wealth invested in asset markets, and $R_{a,t+1}$ denotes the unobservable gross return on the total wealth portfolio. This asset delivers aggregate consumption as its dividends each period. Epstein and Zin (1989) show that the logarithm of the intertemporal marginal rate of substitution (IMRS) is given by

$$m_{t+1} = \theta \ln(\delta) - \frac{\theta}{\psi} g_{t+1} - (1-\theta) r_{a,t+1}, \tag{8}$$

where g_{t+1} denotes the logarithm of aggregate consumption growth and $\ln R_{a,t+1} = r_{a,t+1}$. Note that the IMRS depends on both consumption growth and on the return from the total wealth portfolio. Recall that $\theta = 1$ under power utility, which brings one back to the standard time-separable IMRS.

C. Solving the Model

The returns on the aggregate wealth and market portfolio are approximated using the analytical solutions found in Campbell and Shiller (1988):

$$r_{a,t+1} = k_0 + k_1 z_{t+1} - z_t + g_{t+1}, \tag{9}$$

$$r_{m,t+1} = k_{0,m} + k_{1,m} z_{m,t+1} - z_{m,t} + g_{d,t+1}, \tag{10}$$

where z_t and $z_{m,t}$ denote the log price-consumption ratio and the log price-dividend ratio.[9] The coefficients k_0, k_1, $k_{0,m}$, and $k_{1,m}$ are functions of the average level of z_t and $z_{m,t}$.[10] Bansal and Yaron (2004) conjecture that z_t and $z_{m,t}$ are linear functions of the 2 state variables x_t and σ_t^2:

$$z_t = A_0 + A_1 x_t + A_2 \sigma_t^2, \tag{11}$$

$$z_{m,t} = A_{m,0} + A_{m,1} x_t + A_{m,2} \sigma_t^2. \tag{12}$$

Using the standard Euler equation together with the macro dynamics, Bansal and Yaron (2004) solve for the A coefficients, which are reported in Appendix A1.[11] Focusing on the price-dividend ratio, the coefficient $A_{m,1}$ measures the impact on price-dividend ratios from changes in expected consumption growth. Valuation ratios rise in response to higher expected economic growth when the EIS exceeds 1 and react more strongly to consumption shocks as the persistence, ρ, increases. Here, $A_{m,2}$ governs the response of the price-dividend ratio to changes in

[9]Bansal, Kiku, and Yaron (2007) show that the approximate analytical solution for the wealth return is close to the numerical solution and delivers similar model implications.

[10]The constants are given by $k_1 = \exp(\bar{z})/(1+\exp(\bar{z}))$, $k_0 = \ln(1+\exp(\bar{z})) - k_1\bar{z}$, where $\bar{z}$ denotes the average price-consumption ratio. Similar expression holds for the price-dividend ratio.

[11]The Appendix is available from the author.

macroeconomic uncertainty. An increase in the variance of growth rates lowers valuation ratios when the EIS exceeds 1 and the effect of volatility shocks is amplified as the persistence of shocks, v_1, increases. Consider the following expression for the innovation to the real pricing kernel, where λ represents market prices of risk:

$$(13) \qquad m_{t+1} - \mathrm{E}_t(m_{t+1}) \quad = \quad -\lambda_\eta \sigma_t \eta_{t+1} - \lambda_\varepsilon \sigma_t \varepsilon_{t+1} - \lambda_w \sigma_w w_{t+1},$$

where $\lambda_\eta = \gamma$, $\lambda_\varepsilon = (1-\theta)k_1 A_1 \varphi_e$, and $\lambda_w = (1-\theta)k_1 A_2$. The crucial feature of the model is that long-run risk, ε, and volatility risk, w, are priced in addition to short-run risk, η. The price of long-run risk, λ_ε, is positive when the agent prefers early resolution of uncertainty and $\psi > 1$. Volatility risk, on the other hand, has a negative price if the agent prefers early resolution of uncertainty and ψ and γ exceed 1. Recall that $\theta = 1$ under power utility, which means that only short-run risk is priced. The logarithm of the nominal pricing kernel is determined by the difference between the real pricing kernel and the inflation rate, $m^{\$}_{t+1} = m_{t+1} - \pi_{t+1}$.

D. Model Implications for Bond Prices

In this section, I derive analytical expressions and analyze model implications for real and nominal bonds. See, for example, Backus and Zin (1994) for more on how to price bonds using the stochastic discount factor. Later, the model is estimated using SMM.

1. Real Bonds

Log prices of real bonds with a maturity of n periods are linear functions of the state variables:

$$(14) \qquad q_{t,n} \quad = \quad D_{0,n} + D_{1,n} x_t + D_{2,n} \sigma_t^2.$$

The n-period continuously compounded yield is denoted $y_{t,n} = -(1/n) q_{t,n}$. Using the Euler equation of the agent, Bansal and Yaron (2000) show that

$$(15) \qquad D_{1,n} \quad = \quad \rho D_{1,n-1} - \frac{1}{\psi},$$

$$(16) \qquad D_{2,n} \quad = \quad v_1 D_{2,n-1} + (\theta - 1) A_2 (k_1 v_1 - 1) + \frac{1}{2}\left(\lambda_\eta^2 + (-\lambda_\varepsilon + \varphi_e D_{1,n-1})^2\right),$$

where $D_{1,0} = D_{2,0} = 0$.[12] The $D_{0,n}$ term is reported in Appendix A2 (available from the author). These loadings determine the response of real bonds to movements in the expected mean and variance of real consumption growth. Here, $D_{1,n}$ is negative and increasingly so with maturity, which means that the price of real bonds decreases in response to higher expected growth. Lowering the EIS amplifies the effect, and increasing the persistence, ρ, makes long bonds react

[12] The Euler equation is given by $q_{t,n} = \mathrm{E}_t[m_{t+1} + q_{t+1,n-1}] + ½\mathrm{Var}_t[m_{t+1} + q_{t+1,n-1}]$.

more strongly. The sign of $D_{2,n}$ depends on the preference parameters in a less straightforward way. However, the term is positive for reasonable values of the risk aversion and the EIS, which implies that bond prices increase as macroeconomic uncertainty increases. The magnitude of the coefficient is increasing in the level of risk aversion and in the maturity n of the bond.

2. Nominal Bonds

Nominal bonds are a function of expected inflation, in addition to the conditional mean and variance of consumption growth. Let nominal bond prices and yields be denoted by superscript \$. The log price of a nominal bond takes the form

$$q^{\$}_{t,n} = D^{\$}_{0,n} + D^{\$}_{1,n}x_t + D^{\$}_{2,n}\sigma^2_t + D^{\$}_{3,n}x^{\pi}_t. \tag{17}$$

The n-period continuously compounded nominal yield is denoted $y^{\$}_{t,n} = -(1/n)\, q^{\$}_{t,n}$. I show in Appendix A2 (available from the author) that the loadings are defined as

$$D^{\$}_{1,n} = \rho D^{\$}_{1,n-1} - \frac{1}{\psi}, \tag{18}$$

$$\begin{aligned} D^{\$}_{2,n} = {} & v_1 D^{\$}_{2,n-1} + (\theta-1)A_2(k_1 v_1 - 1) \\ & + \frac{1}{2}\left(\lambda^2_\eta + \left(-\lambda_\varepsilon + \varphi_e D^{\$}_{1,n-1} + \delta_2 D^{\$}_{3,n-1}\right)^2 \right. \\ & \left. + \left(D^{\$}_{3,n-1}\delta_3\right)^2 + \delta^2_1\right), \end{aligned} \tag{19}$$

$$D^{\$}_{3,n} = D^{\$}_{3,n-1}\rho_\pi - 1, \tag{20}$$

where $D^{\$}_{1,0} = D^{\$}_{2,0} = D^{\$}_{3,0} = 0$.[13] The $D^{\$}_{0,n}$ term is reported in the Appendix (available from the author). The new term, $D^{\$}_{3,n}$, governs the response of nominal bonds to inflation. The term is negative and increasingly so for longer maturities. Furthermore, the introduction of inflation affects the loading on volatility, since the last term in expression (19) is different compared to real bonds. The term $(-\lambda_\varepsilon + \varphi_e D^{\$}_{1,n-1} + \delta_2 D^{\$}_{3,n-1})^2$ determines whether nominal bonds are a hedge against macroeconomic uncertainty or not. A negative relation between inflation and consumption growth, $\delta_2 < 0$, decreases the value of the squared expression, which may lead to a drop in the price of nominal bonds as macroeconomic volatility increases.

III. Data and Estimation of Model

A. Data

Quarterly aggregate U.S. consumption data on nondurables and services are collected from the Bureau of Economic Analysis for the period 1952:2–2007:4. Inflation is computed as in Piazzesi and Schneider (2006) using the price index

[13]The Euler equation used is $q^{\$}_{t,n} = \mathrm{E}_t[m_{t+1} - \pi_{t+1} + q^{\$}_{t+1,n-1}] + ½\mathrm{Var}_t[m_{t+1} - \pi_{t+1} + q^{\$}_{t+1,n-1}]$.

that corresponds to the consumption data. Appendix A3 (available from the author) reports that this inflation measure is less volatile and more persistent than the CPI. Value-weighted market returns (NYSE/AMEX) are retrieved from the Center for Research in Security Prices (CRSP). Nominal interest rates are collected from the Fama-Bliss (1987) file in CRSP and from the Web site of J. Huston McCulloch (http://www.econ.ohio-state.edu/jhm/jhm.html). Dividend growth is computed using monthly CRSP returns including and excluding dividends as in, for example, Bansal, Dittmar, and Lundblad (2005). Quarterly dividends, D_t, are formed by summing monthly dividends. Due to the strong seasonality of dividend payments, I use a 4-quarter moving average of dividend payments, $\bar{D}_t = (D_t + D_{t-1} + D_{t-2} + D_{t-3})/4$. Real dividend growth rates are found by taking the log 1st difference of $\bar{D}_t$ and deflating using the constructed inflation series.

Table 1 reports observed macro moments. Consumption growth exhibits a quarterly volatility of 0.47% over the sample period, which is less than the

TABLE 1
Macro Moments

Table 1 presents unconditional moments of observed and model-implied macro data. All moments are based on quarterly data. Population statistics are based on a simulation of 150,000 months. Medians and percentiles for the model are computed at over 2,000 simulations each using 669 months aggregated to 223 quarters. AC(k) denotes the autocorrelation for k lags. AC with 4 lags is reported for dividend growth, since it is constructed using a moving average of dividend payments from time t to $t-3$. Standard errors (SE) are computed as in Newey and West (1987) using 12 lags. The sample period is 1952:2–2007:4.

	Sample		Model			
Macro Moments	Moment	SE	Pop.	Median	5%	95%
Panel A. Consumption Growth, g						
Mean	0.81	(0.05)	0.81	0.81	0.63	0.99
Std. dev.	0.47	(0.03)	0.44	0.37	0.26	0.53
AC1	0.35	(0.06)	0.38	0.29	0.20	0.42
AC2	0.18	(0.06)	0.21	0.10	0.00	0.27
Panel B. Dividend Growth, g_d						
Mean	0.51	(0.21)	1.01	1.00	0.44	1.55
Std. dev.	1.53	(0.42)	1.48	1.24	0.87	1.80
AC4	0.16	(0.13)	0.16	0.06	−0.08	0.22
Panel C. Inflation, π						
Mean	0.92	(0.13)	0.92	0.92	0.54	1.29
Std. dev.	0.62	(0.12)	0.67	0.51	0.34	0.83
AC1	0.83	(0.05)	0.81	0.74	0.56	0.87
AC2	0.78	(0.06)	0.78	0.69	0.49	0.84
Panel D. Correlations						
g and g_d	0.17	(0.06)	0.20	0.10	−0.06	0.26
g and π	−0.35	(0.07)	−0.32	−0.23	−0.43	−0.04
g_d and π	−0.19	(0.08)	−0.28	−0.20	−0.39	−0.02

volatility of inflation, 0.62%, and the volatility of dividend growth, 1.53%. Both consumption growth and inflation display statistically significant autocorrelation coefficients for 1 and 2 lags. However the persistence of inflation is significantly higher. I report the 4th-order autocorrelation coefficient for dividend growth, since the moving average procedure automatically induces positive autocorrelation for up to 3 lags. The correlation between consumption growth and dividend growth is positive 0.17, while correlations between real growth rates and inflation are negative, −0.35 for consumption growth and −0.19 for dividend growth.

B. Estimation

Reported aggregated consumption measures consumption expenditures over a period rather than at a fixed point in time, which gives rise to a temporal aggregation effect.[14] To account for temporal aggregation, the decision interval of the representative agent is assumed to be monthly, while targeted data consist of quarterly moments of observed data. Quarterly moments implied by the model are computed by aggregating monthly observations. Appendix A4 (available from the author) describes how the endogenous coefficients k_0, k_1, $k_{0,m}$, and $k_{1,m}$ are solved in the estimation.

SMM is an estimation method that accounts for time-aggregation effects and allows for simulation of long samples. The procedure is described in Lee and Ingram (1991) and Duffie and Singleton (1993) and aims at minimizing the distance between actual sample moments and simulated model moments. Appendix A5 (available from the author) describes the SMM procedure in detail.

Parameters governing the macro dynamics together with the risk-aversion parameter and the EIS are estimated using moments of macro data and asset price data. Restrictions are imposed in the estimation to rule out unit roots and ensure stationarity and ergodicity of simulated macro variables. Arguably, asset prices contain important information about future economic prospects and should therefore be useful for estimating the macro dynamics. See, for example, Backus, Routledge, and Zin (2010) and Section IV.D for evidence that asset prices predict future economic output. Also, cross moments between asset prices and macro variables are important to capture for any macroeconomic model that tries to explain asset prices (e.g., Cochrane and Hansen (1992)). In an earlier version of this paper, I estimated the model using only macro data. This estimation turned out to be inefficient and produced large standard errors for important variables of the model, in particular for the volatility dynamics. Incorporating asset pricing information increases efficiency substantially. As is shown below, incorporating asset pricing moments does not materially affect the model's ability to fit key macro moments.

I calibrate the discount factor to 0.9992, which is close to the value estimated in Bansal, Gallant, and Tauchen (2007). The estimation makes use of a rich set of moment conditions in order to identify parameters of the model. Table 2 describes the moment conditions, which can be divided into 3 sets. The 1st set contains 11 macro moments that capture the 1st and 2nd (uncentered) moments of consumption growth, dividend growth, and inflation. Matching the means together with the uncentered 2nd moments implies matching the unconditional volatility of each variable. I also include the expected value of lagged variables in order to match autocorrelations. I consider 1 and 2 lags for consumption growth and inflation. The 2nd set of moment conditions consists of 8 asset price moments aimed at capturing the mean and variance of excess stock returns, 3-month nominal interest rates, the difference between 5-year and 3-month nominal interest rates, and log price-dividend ratios. The 3rd set consists of 3 moments that capture evidence that

[14]Working (1960) shows that the time averaging of an IID process automatically induces positive autocorrelation and produces a less volatile series compared to the original one.

TABLE 2

Moment Conditions

Table 2 lists the moment conditions used in the estimation. Here, ξ_{t+1} refers to the error term from an AR(1) process fitted to quarterly consumption growth within the model and in the data and which is used to match the evidence that price-dividend ratios predict future volatility of consumption growth.

Moment Conditions	
Panel A. Macro Moments	
Mean of consumption growth	$E(g_t)$
Mean of inflation	$E(\pi_t)$
Mean of dividend growth	$E(g_{d,t})$
Mean of squared consumption growth	$E(g_t^2)$
Mean of squared inflation	$E(\pi_t^2)$
Mean of squared dividend growth	$E(g_{d,t}^2)$
Mean product of consumption growth at time t+1 and t	$E(g_{t+1}g_t)$
Mean product of consumption growth at time t+2 and t	$E(g_{t+2}g_t)$
Mean product of inflation at time t+1 and t	$E(\pi_{t+1}\pi_t)$
Mean product of inflation at time t+2 and t	$E(\pi_{t+2}\pi_t)$
Mean product of consumption growth and inflation	$E(g_t\pi_t)$
Panel B. Asset Price Moments	
Mean equity excess return	$E(r_{m,t}^{\$} - y_{t,3m}^{\$})$
Mean nominal 3-month interest rate	$E(y_{t,3m}^{\$})$
Mean nominal yield spread	$E(y_{t,5y}^{\$} - y_{t,3m}^{\$})$
Mean log price-dividend ratio	$E(\mathrm{PD}_t)$
Mean squared equity excess return	$E((r_{m,t}^{\$} - y_{t,3m}^{\$})^2)$
Mean squared nominal 3-month interest rate	$E((y_{t,3m}^{\$})^2)$
Mean squared nominal yield spread	$E((y_{t,5y}^{\$} - y_{t,3m}^{\$})^2)$
Mean squared log price-dividend ratio	$E(\mathrm{PD}_t^2)$
Panel C. Predictability	
Mean squared residual	$E(\xi_{t+1}^2)$
Mean product of squared residual and the log price-dividend ratio	$E(\xi_{t+1}^2\mathrm{PD}_t)$
Mean product of consumption growth and the nominal yield spread	$E(g_{t+1}(y_{t,5y}^{\$} - y_{t,3m}^{\$}))$

asset prices predict future macroeconomic variables. First, price-dividend ratios predict future consumption growth volatility negatively in the data. This is used in the estimation by including the cross product of PD_t and the consumption growth volatility 1 quarter ahead. Volatility is measured as the squared residual stemming from an AR(1) process fitted to consumption growth. Matching the cross product together with expected squared residuals and the expected level and volatility of log price-dividend ratios implies matching the regression coefficient from regressing future consumption growth volatility onto today's price-dividend ratio. Second, the nominal term spread predicts future consumption growth positively in the data. This is used as a restriction by including the cross-moment between the yield spread at time t and consumption growth at time $t+1$.

There are in total 16 parameters to estimate and 22 moments to match, which gives 6 overidentifying restrictions. One could in principle introduce even more moment conditions, but I have chosen to limit the number of restrictions and focus on the most fundamental macro and asset pricing moments. The 2 predictive regressions are used as moment conditions, since they directly impose identifying restrictions on the EIS, a parameter that traditionally has been difficult to estimate precisely. Section IV shows why these moment conditions help to identify the EIS. I use the optimal weighting matrix throughout the paper.

Table 3 presents the estimation results. Shocks to expected consumption growth are estimated to be highly persistent, with ρ equal to 0.9957. The long-run risk component, φ_e, is estimated to 0.0248 with a standard error of 0.0140. The persistence of volatility shocks, v_1, is also estimated to be high, 0.9968. The

TABLE 3

Estimated Parameters

Table 3 presents results from estimating the parameters of the model using simulated method of moments (SMM). The sample period is 1952:2–2007:4. The sample covariance matrix in the SMM procedure is computed as in Newey and West (1987) using 12 lags. Standard errors (SE) are given in parentheses, and *c* refers to a calibrated parameter.

Estimated Parameters		Estimate	SE
Panel A. Real Parameters			
Mean of consumption growth	μ	0.00268	(0.00014)
Mean of dividend growth	μ_d	0.00336	(0.00066)
Persistence of expected consumption growth	ρ	0.9957	(0.0032)
Volatility of long-run consumption shocks	φ_e	0.0248	(0.0140)
Persistence of volatility	v_1	0.9968	(0.0026)
Volatility of volatility shocks	$\sigma_w \times 10^{-5}$	0.0691	(0.0524)
Mean of volatility	σ	0.0012	(0.0036)
Loading of dividend growth on expected consumption growth	ϕ	2.85	(0.79)
Volatility of dividend shocks	φ_d	3.51	(0.51)
Panel B. Inflation Parameters			
Mean of inflation	μ_π	0.00305	(0.00025)
Volatility of short-run inflation shocks	δ_1	0.5840	(3.3720)
Persistence of expected inflation	ρ_π	0.9851	(0.0026)
Impact of long-run consumption shocks on expected inflation	δ_2	−0.1254	(0.0250)
Volatility of long-run inflation shocks	δ_3	0.0475	(0.0652)
Panel C. Preference Parameters			
Discount factor	δ	0.9992	*c*
Risk aversion	γ	6.78	(1.61)
Elasticity of intertemporal substitution	ψ	2.51	(0.74)
Panel D. SMM Statistics			
$\chi^2(6)$		19.35	
p-value		0.0036	

parameter governing the volatility of volatility, σ_w, is estimated to 0.0691×10^{-5}, and the mean of the volatility process, σ, is estimated to 0.0012. Compared to Bansal and Yaron (2004), the estimated persistence of long-run shocks is higher, while φ_e is lower. Also, the persistence of volatility shocks is higher than in the original long-run risk model. My estimation suggests a half-life of 216 months for volatility shocks compared to 33 months in Bansal and Yaron (2004). Furthermore, the unconditional volatility and the volatility of volatility are estimated to be lower than what is commonly used in the long-run risk literature. The process for volatility is assumed to be normally distributed, which allows for tractable analytical solutions but means it can take on negative values. To avoid this, I replace negative values in the simulation with a number close to 0. Shocks to expected inflation are also estimated to be highly persistent, with ρ_π equal to 0.9851. The parameter governing the sign of the correlation between consumption growth and inflation, δ_2, is estimated to -0.1254. The modeling of inflation is in reduced form, which implies that δ_2 has nothing to say about the underlying sources of the inflation-growth relation. It is worth pointing out that the negative correlation observed in the data is largely due to the stagflation period in the 1970s.

Endogenizing inflation in terms of supply versus demand shocks or via a monetary policy channel is interesting in terms of avenues of future research but outside the scope of this paper. The effect of long-run shocks to inflation is governed by δ_3 and is estimated to 0.0475.

Panel C of Table 3 reports the risk-aversion estimate of 6.78, which is lower than the commonly calibrated value of 10. The EIS is estimated to 2.51 with a standard error of 0.74. This implies an EIS that is significantly different from the inverse of the risk-aversion coefficient that is the case with power utility.[15] The estimated preference parameters imply that the representative agent prefers early resolution of uncertainty, that long-run risk has a positive price, $\lambda_\varepsilon > 0$, and that volatility risk has a negative price, $\lambda_w < 0$.

An EIS of 2.51 stands in contrast to values close to 0 that have been found by regressing consumption growth onto the real rate (e.g., Hall (1988), Campbell (1999)). My results suggest that incorporating more model-based restrictions when estimating the EIS leads to significantly different results compared to the classical regressions. The estimation generates a χ^2 statistic of model fit of 19.35 with a p-value of 0.0036. Despite being statistically rejected, the model comes close to matching several key moments of asset markets as described later.

Interestingly, estimating the model using the CPI as inflation measure fails to reject the model, generating a χ^2 statistic of 8.93 and a p-value of 0.18 (see Appendix A3 (available from the author)). The main difference is the stronger impact of long-run inflation shocks, δ_3, which is estimated to 0.0820 compared to 0.0475 in the original estimation. This reflects the higher volatility and lower persistence of CPI inflation that helps in particular to better match the volatility of nominal interest rates. Considering that χ^2 tests tend to overreject, the model seems to provide a reasonable fit to the data.

The estimated parameter values are used to simulate the model, and Table 1 reports the distribution of simulated macro moments. The population moments of the model all lie close to their sample values. For consumption growth, the population and median values of the volatility are slightly lower than in the data, while the population autocorrelation coefficients are slightly higher compared to the data. The mean of dividend growth is somewhat overestimated, but the sample mean lies within the simulated 5th and 95th percentiles. The volatility and persistence of dividend growth lie close to their sample values. The model also matches the inflation moments and the macro correlations closely.

IV. Implications for Asset Prices

This section describes the dynamics of asset prices, implied by the estimated parameters.

[15]For comparison, Bansal, Gallant, and Tauchen (2007) estimate the risk aversion to 7 while calibrating the EIS to 2. Bansal, Kiku, and Yaron (2007) estimate the risk aversion to 10 and the EIS to 2.43. Chen, Favilukis, and Ludvigson (2008) estimate a model with recursive preferences and a general specification for consumption growth. They estimate the risk aversion to lie in the range of 17–60 and the EIS to be above 1.

A. Real Term Structure

Prices on real bonds are negatively related to long-run risk (i.e., $D_{1,n} < 0$), which leads to higher yields in response to positive shocks to expected consumption growth. Real yields are therefore procyclical and provide a hedge against a drop in consumption growth. The loadings on volatility risk, $D_{2,n}$, are positive, indicating that real bonds act as a hedge against positive shocks to macroeconomic uncertainty, with long bonds being more sensitive than short bonds. Accordingly, real bonds are subject to negative risk premiums, as they provide insurance against bad times. Let $h_{t+1,n} = q_{t+1,n-1} - q_{t,n}$ denote the 1-period log holding period return on a bond with a maturity of n periods. The risk premium can be written as

$$\text{(21)} \qquad \begin{aligned} \mathrm{E}_t(h_{t+1,n} - r_{f,t}) + \frac{1}{2}\mathrm{Var}_t(h_{t+1,n}) &= -\mathrm{Cov}_t(m_{t+1}, h_{t+1,n}), \\ &= \lambda_\varepsilon \varphi_e D_{1,n-1}\sigma_t^2 + \lambda_w D_{2,n-1}\sigma_w^2, \end{aligned}$$

where the variance term on the left-hand side is a Jensen's inequality term. The risk premium depends on the market prices of risk and the loadings on long-run and volatility risks, while being independent of short-run risks. A positive price of long-run risks and a negative value of $D_{1,n-1}$ imply a negative risk premium. Similarly, a negative price of volatility risk and a positive value of $D_{2,n-1}$ also imply a negative expected excess return.[16] The stochastic volatility of consumption growth, σ_t^2, gives rise to a time-varying risk premium, where an increase in volatility lowers risk premiums. Both λ_ε and λ_w equal 0 under power utility, which implies constant risk premiums (ignoring the Jensen's term).

Next, consider the unconditional slope of the real yield curve measured as the long rate (60 months) minus the short rate (3 months):

$$\text{(22)} \qquad \mathrm{E}(y_{t,60} - y_{t,3}) = \left(\frac{D_{0,3}}{3} - \frac{D_{0,60}}{60}\right) + \left(\frac{D_{2,3}}{3} - \frac{D_{2,60}}{60}\right)\sigma^2,$$

which is mainly determined by the average level of uncertainty in the economy and the difference in loadings across maturities on volatility shocks. A higher sensitivity of long yields to volatility shocks contributes to a negative slope. Table 4 reports a downward-sloping real yield curve in the model, which is supported by Evans (1998) and Piazzesi and Schneider (2006), who document a negative slope for U.K. index-linked bonds. Data for U.S. index-linked bonds indicate a positive slope, but the time series only dates back to 1997, and the market was illiquid at the beginning of the sample. The model also produces a downward-sloping term structure of volatility and highly persistent real yields (not shown in the table). This is consistent with data from both the United States and the United Kingdom.

[16]Bansal and Yaron (2000) briefly mention that their model generates negative risk premiums for real bonds, but they do not elaborate further on the issue.

TABLE 4
Term Structure of Real Interest Rates

Table 4 presents the model-implied term structure of real interest rates. All yields are in annualized percentages. Reported statistics are population statistics from a simulation of 150,000 months.

	Model	
Maturity	Pop.	Std. Dev.
3m	0.65	0.56
1y	0.63	0.56
2y	0.57	0.56
3y	0.52	0.56
4y	0.46	0.55
5y	0.41	0.55

B. Nominal Term Structure

Consider the innovation to nominal yields:

$$y_{t+1,n}^{\$} - \mathrm{E}_t\left(y_{t+1,n}^{\$}\right) = -\frac{1}{n}\left(\left(D_{1,n}^{\$}\varphi_e + D_{3,n}^{\$}\delta_2\right)\sigma_t\varepsilon_{t+1} + D_{2,n}^{\$}\sigma_w w_{t+1} + D_{3,n}^{\$}\delta_3\sigma_t\varepsilon_{t+1}^{\pi}\right). \tag{23}$$

The response of nominal rates to long-run consumption risks, ε_{t+1}, depends on $D_{1,n}^{\$}\varphi_e$ and $D_{3,n}^{\$}\delta_2$. Setting $\delta_2 = 0$ implies that both real and nominal yields increase in response to a positive expected growth shock. However, δ_2 is estimated to be negative, which imposes a wedge between real and nominal yields. A positive shock to expected growth has 2 effects on nominal yields. First, yields rise through a real channel governed by $D_{1,n}^{\$}\varphi_e$. Second, yields drop through a nominal channel $D_{3,n}^{\$}\delta_2$, since a positive growth shock leads to a drop in inflation. The estimated value of $\delta_2 = -0.1254$ makes the 2nd effect dominate, which makes nominal yields become countercyclical and nominal bond returns procyclical. Recall that the volatility loading for nominal bonds is different than for real bonds. Long yields now rise as macroeconomic uncertainty increases, yielding low bond returns. As a result, long-term nominal bonds do not provide insurance against bad times and are subject to positive risk premiums.

Let $h_{t+1,n}^{\$} = q_{t+1,n-1}^{\$} - q_{t,n}^{\$}$ denote the 1-period log holding period return on a nominal bond with a maturity of n periods. The risk premium can be written as

$$\begin{aligned} \mathrm{E}_t(h_{t+1,n}^{\$} - r_{f,t}) + \frac{1}{2}\mathrm{Var}_t(h_{t+1,n}^{\$}) &= -\mathrm{Cov}_t(m_{t+1}^{\$}, h_{t+1,n}^{\$}), \\ &= \lambda_{\varepsilon}(\varphi_e D_{1,n-1}^{\$} + D_{3,n-1}^{\$}\delta_2)\sigma_t^2 \\ &\quad + \lambda_w D_{2,n-1}^{\$}\sigma_w^2. \end{aligned} \tag{24}$$

Given a sufficiently negative correlation between consumption growth and inflation, governed by δ_2, risk premiums increase when consumption growth volatility increases. This highlights a key difference between risk premiums on real versus nominal bonds. While risk premiums on real bonds decrease in response to higher volatility, risk premiums on nominal bonds increase.

The average nominal slope can be written as in equation (22) but with nominal yield loadings. Since short-term bonds provide a better hedge against volatility shocks than long-term bonds, the slope of the yield curve loads positively on the level of volatility as opposed to negatively for real bonds. Implications for the nominal yield curve are reported in Table 5. The model comes close to matching the slope of the nominal yield curve and its volatility. The model generates a

TABLE 5

Term Structure of Nominal Interest Rates

Table 5 reports the model-implied and observed nominal term structure of interest rates using quarterly observations. All yields are in annualized percentages. Reported model statistics are population statistics from a simulation of 150,000 months. The sample period is 1952:2–2007:4.

	Sample		Model	
Maturity	Mean	Std. Dev.	Pop.	Std. Dev.
3m	5.28	2.85	4.36	2.23
1y	5.52	2.88	4.49	2.07
2y	5.72	2.84	4.65	1.87
3y	5.88	2.78	4.80	1.70
4y	6.00	2.75	4.94	1.56
5y	6.08	2.71	5.07	1.44
5y–3m	0.80	1.02	0.71	0.94

positive slope of 71 basis points (bp) compared to 80 bp in the data. The volatility of the yield spread is 0.94% in the model compared to 1.02% in the data. A lower value of δ_2 translates into a higher inflation risk premium on nominal bonds and therefore a steeper yield curve. The model also generates a downward-sloping term structure of volatility and highly persistent yields as they inherit the persistence from the state variables. The volatility of model yields is somewhat lower than in the data. The estimated model in Appendix A3 (available from the author), which uses CPI as an inflation measure, is able to generate a higher volatility of interest rates, since CPI exhibits higher volatility than the price index corresponding to the consumption series.[17]

C. Equity

Since the model implications for equity are discussed in Bansal and Yaron (2004), I choose to discuss the implications only briefly. The simulated and observed unconditional equity moments are reported in Table 6. Overall, the model matches data well. The model produces a mean and volatility of the equity risk premium of 1.53% and 7.31% versus 1.40% and 7.78% in the data, respectively. Also, the moments for log price-dividend ratios lie close to the data, with a model-generated mean and volatility of 3.55 and 0.39 versus 3.46 and 0.35 in the data, respectively.

D. Predictability

This section explores the model implications for predictability of asset returns and future macroeconomic conditions.

[17]For brevity, asset pricing implications for this estimation are not reported but are available from the author.

TABLE 6

Equity

Table 6 presents model-implied and observed moments for equity. All moments are reported on a quarterly basis. Population coefficients are obtained from simulating 1 sample of 150,000 months. The sample period is 1952:2–2007:4.

	Sample		Model
Moments	Moment	SE	Pop.
$E(r_m - r_f)$	1.40	0.51	1.53
$\sigma(r_m - r_f)$	7.78	0.59	7.31
E(PD)	3.46	0.05	3.55
σ(PD)	0.35	0.03	0.39
AC1(PD)	0.97	0.02	0.98

1. The Expectations Hypothesis

The expectations hypothesis can be expressed in different forms (e.g., Cox, Ingersoll, and Ross (1981), Campbell, Lo, and MacKinlay (1997)). One version states that log excess holding period returns for bonds differ across maturities but are constant through time. Evidence documented in Fama and Bliss (1987) and Campbell and Shiller (1991) indicate that risk premiums on U.S. nominal bonds in fact vary over time. Evans (1998), (2003) document time-varying risk premiums also for real bonds using U.K. data.

Fama and Bliss (1987) use the insight that the forward-spot spread must predict either future bond excess returns or changes in short rates and run the following classical regression, $hx^{\$}_{t+1,n} = \alpha_n + \beta_n(f^{\$}_{t,n} - y^{\$}_{t,1}) + \epsilon_{t+1,n}$, where $hx^{\$}_{t+1,n} = q^{\$}_{t+1,n-1} - q^{\$}_{t,n} - y^{\$}_{t,1}$ denotes the annual log excess return and where $f^{\$}_{t,n} = q^{\$}_{t,n-1} - q^{\$}_{t,n}$ denotes the log forward rate. The expectations hypothesis suggests that risk premiums are constant so $\beta_n = 0$.

Table 7 indicates that the expectations hypothesis is rejected both in the data and in the model. An increase in the volatility of consumption growth raises both forward rates and risk premiums, generating a positive comovement between forward-spot spreads and expected excess returns in the model. The model-implied

TABLE 7

Fama and Bliss Regressions

Table 7 presents results from testing the expectations hypothesis for real and nominal interest rates by running the Fama and Bliss (1987) regression: $h^{\$}_{t+1,n} - y^{\$}_{t,1} = \alpha_n + \beta_n(f^{\$}_{t,n} - y^{\$}_{t,1}) + \epsilon_{t+1,n}$ for nominal bonds and the same for real bonds without the \$ superscript. Log forward rates are defined as $f^{\$}_{t,n} = q^{\$}_{t,n-1} - q^{\$}_{t,n}$, where q denotes the log bond price. The forecast horizon is 1 year and n is 2–5 years. The t-stat values correspond to H_0: $\beta_n = 0$. Standard errors are computed as in Newey and West (1987) using 12 lags. Pop betas refer to betas obtained from simulating 1 sample of 150,000 months. The sample period is 1952:2–2007:4.

	Sample		Model	
Maturity	$\beta^{\$}_{n,m}$	t-Stat	Pop $\beta^{\$}_{n,m}$	Pop $\beta_{n,m}$
2y	0.83	3.79	0.32	1.11
3y	1.06	3.67	0.32	1.08
4y	1.29	3.61	0.31	1.05
5y	1.00	2.32	0.31	1.01

regression coefficients are, however, smaller than in the data. The model also rejects the expectations hypothesis for real bonds with slope coefficients that are larger than for nominal bonds.

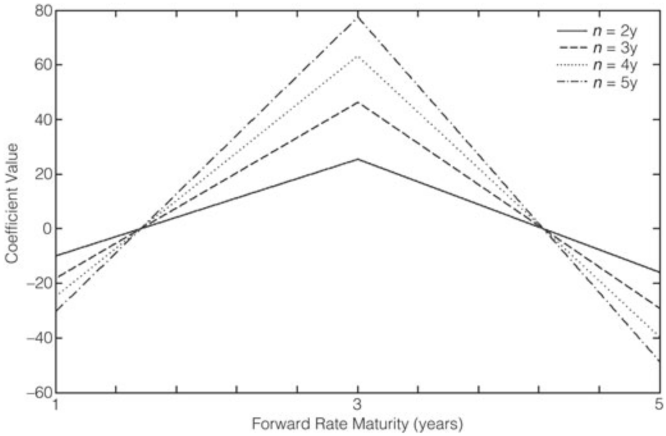
Cochrane and Piazzesi (2005) show that predicting excess returns with 5 forward rates generates high explanatory power and produces tent-shaped regression coefficients. I test whether the model can match their evidence by running the same regression but with 3 forward rates as explanatory variables, $hx^{\$}_{t+1,n} = \alpha_n + \beta_{1,n} y^{\$}_{t,1} + \beta_{2,n} f^{\$}_{t,3} + \beta_{3,n} f^{\$}_{t,5} + \epsilon_{t+1,n}$.

Figure 1 shows that the model is capable of generating a similar tent shape as in Cochrane and Piazzesi (2005), albeit with regression coefficients that are

FIGURE 1

Cochrane and Piazzesi Regressions

Figure 1 displays model-implied regression coefficients from regressing annual excess returns on 2-, 3-, 4-, and 5-year bonds onto 1-, 3-, and 5-year forward rates. Coefficients are obtained from a simulated sample of 150,000 months.



larger in magnitude. Table 8 documents model-implied R^2s in the region of 4%–5% compared to 22%–26% in the data. Overall, the long-run risk model is capable of generating predictability of bond returns through a time-varying volatility of consumption growth, but deviations from the expectations hypothesis are smaller compared to the data. It is possible to generate stronger predictability within the model but at the cost of generating counterfactual macro implications.

Dai and Singleton (2002) show that affine models with Gaussian factors and time-varying market prices of risk perform much better in generating predictability of bond returns than models with time-varying volatility and constant market prices of risk. The long-run risk model falls within the affine class but relies on time-varying volatility to generate predictability. Perhaps incorporating time-varying market prices of risk as in Le and Singleton (2010) would improve the model's ability to generate bond return predictability. It is important to note that the long-run risk model incorporates a large number of model restrictions that are absent when estimating typical latent-factor Gaussian models.

TABLE 8

Cochrane and Piazzesi Regressions

Table 8 presents the explanatory power of the Cochrane and Piazzesi (2005) regression: $h^{\$}_{t+1,n} - y^{\$}_{t,1} = \alpha_n + \beta_{1,n} y^{\$}_{t,1} + \beta_{2,n} f^{\$}_{t,3} + \beta_{3,n} f^{\$}_{t,5} + \epsilon_{t+1,n}$ for nominal bonds and the same for real bonds without the \$ superscript. Log forward rates are defined as $f^{\$}_{t,n} = q^{\$}_{t,n-1} - q^{\$}_{t,n}$, where q denotes the log bond price. The forecast horizon is 1 year and n is 2–5 years. Population values are obtained from simulating 1 sample of 150,000 months. The sample period is 1952:2–2007:4.

	Sample	Model	
Maturity	$R^2_{\$}$	Pop $R^2_{\$}$	Pop R^2
2y	0.22	0.04	0.05
3y	0.23	0.04	0.05
4y	0.26	0.04	0.04
5y	0.23	0.04	0.04

2. Predicting with the Yield Spread

The yield spread's ability to predict economic growth positively is well established (e.g., Stock and Watson (1989), Estrella and Hardouvelis (1991), and Ang et al. (2006)). Table 9 reports the positive slope coefficients found in the data. The results suggest that the yield curve is a short-term predictor of consumption growth, since the explanatory power peaks at 12% for the 1-year horizon and then vanishes. The model is able to generate similar predictability up to a 1-year horizon but fails to match the drop in predictability for longer horizons. Key for generating positive regression coefficients is a high EIS together with a negative correlation between growth and inflation. See Section IV.D.4 for a deeper discussion.

TABLE 9

Predicting Consumption Growth with the Nominal Yield Spread

Table 9 presents results from predicting consumption growth using the nominal yield spread. The following regression is run: $g_{t:t+k} = \alpha_k + \beta_k (y^{\$}_{t,5y} - y^{\$}_{t,3m}) + \epsilon_{t+k}$. The forecast horizons, k, are 1, 4, 12, and 20 quarters. Consumption growth and yields are expressed in annualized percentages. Population values are obtained from simulating 1 sample of 150,000 months. Standard errors are computed as in Newey and West (1987) using lags of 2 × horizon. The sample period is 1952:2–2007:4.

	Sample			Model	
Quarters, k	β_k	t-Stat	R^2_{adj}	Pop β_k	Pop R^2_{adj}
1	0.44	2.72	0.05	0.52	0.08
4	0.43	2.67	0.12	0.51	0.16
12	0.10	0.68	0.01	0.49	0.22
20	−0.02	−0.24	0.00	0.46	0.23

The nominal yield curve is also a predictor of future stock returns, albeit a weaker predictor than for economic growth. Table 10 indicates that the yield curve predicts stock returns positively, with an R^2 in the range of 1%–6%. However, the slope coefficients are only significant for the 3-year horizon. The same table shows that the model comes close to matching both the coefficients and the explanatory power of the regressions.

TABLE 10

Predicting Excess Stock Returns with the Nominal Yield Spread

Table 10 presents results from predicting excess stock returns using the nominal yield spread. The following regression is run: $r^{\$}_{t:t+k} - y^{\$}_{t,k} = \alpha_k + \beta_k(y^{\$}_{t,5y} - y^{\$}_{t,3m}) + \epsilon_{t+k}$, where the dependent variable is the nominal stock return for the period t: $t+k$ minus the nominal yield at time t with a maturity of k quarters. The forecast horizons, k, are 1, 4, 12, and 20 quarters. Excess returns and yields are expressed in annualized percentages. Population values are obtained from simulating 1 sample of 150,000 months. Standard errors are computed as in Newey and West (1987) using lags of 2 × horizon. The sample period is 1952:2–2007:4.

	Sample			Model	
Quarters, k	β_k	t-Stat	R^2_{adj}	Pop β_k	Pop R^2_{adj}
1	3.53	1.66	0.01	1.81	0.01
4	2.89	1.77	0.03	1.60	0.01
12	1.14	2.00	0.01	1.21	0.02
20	1.41	1.77	0.06	0.98	0.03

3. Predicting with the Price-Dividend Ratio

A voluminous literature has documented the ability of the price-dividend ratio to predict future excess stock returns.[18] Table 11 documents that price-dividend ratios predict excess returns negatively with an R^2 that increases with horizon and peaks at 18% for the 5-year horizon. The same predictive regressions are run inside the model and are found to generate similar regression coefficients as in the data. However, the explanatory power is smaller inside the model.

TABLE 11

Predicting Excess Stock Returns with the Price-Dividend Ratio

Table 11 presents results from predicting excess stock returns using the log price-dividend ratio. The following regression is run: $r^{\$}_{t:t+k} - y^{\$}_{t,k} = \alpha_k + \beta_k \text{PD}_t + \epsilon_{t+k}$, where the dependent variable is the nominal stock return for the period t: $t+k$ minus the nominal yield at time t with a maturity of k quarters. The forecast horizons, k, are 1, 4, 12, and 20 quarters. Excess returns are in annualized percentages. Population values are obtained from simulating 1 sample of 150,000 months. Standard errors are computed as in Newey and West (1987), using lags of 2 × horizon. The sample period is 1952:2–2007:4.

	Sample			Model	
Quarters, k	β_k	t-Stat	R^2_{adj}	Pop β_k	Pop R^2_{adj}
1	−10.9	−1.88	0.01	−5.61	0.01
4	−10.4	−2.03	0.05	−5.13	0.02
12	−7.8	−2.19	0.12	−4.04	0.04
20	−7.4	−3.25	0.18	−3.26	0.06

Bansal, Khatchatrian, and Yaron (2005) show empirically that price-dividend ratios are negatively related to the volatility of consumption growth. This relation is also shown to be present in countries outside the United States. Table 12 documents the negative relation and indicates that the explanatory power rises with horizon, reaching 23% for a 5-year horizon. The model does well in matching the data, generating regression coefficients in the range of −0.71 to −0.56, with R^2s between 5% and 23%.

[18]An incomplete list is: Campbell and Shiller (1988), Fama and French (1988), Stambaugh (1999), Lewellen (2004), and Ang and Bekaert (2006).

TABLE 12

Predicting Consumption Growth Volatility with the Price-Dividend Ratio

Table 12 presents results from predicting consumption growth volatility using the log price-dividend ratio. The following regression is run: $\log \sum_{j=1}^{k} |\xi_{t+j}| = \alpha_k + \beta_k \mathrm{PD}_t + \epsilon_{t+k}$, where the residuals in the dependent variable stem from an AR(1) process fitted to quarterly consumption growth. The forecast horizons, k, are 1, 4, 12, and 20 quarters. Population values are obtained from simulating 1 sample of 150,000 months. Standard errors are computed as in Newey and West (1987) using lags of 2 × horizon. The sample period is 1952:2–2007:4.

	Sample			Model	
Quarters, k	β_k	t-Stat	R^2_{adj}	Pop β_k	Pop R^2_{adj}
1	−0.55	−2.88	0.03	−0.71	0.05
4	−0.53	−4.18	0.14	−0.69	0.18
12	−0.51	−4.33	0.24	−0.61	0.23
20	−0.43	−2.76	0.23	−0.56	0.23

4. Predictability and the EIS

This section shows that the established relation between yield spreads and future economic growth has direct implications for the value of the EIS and therefore imposes identifying restrictions on the parameter. The covariance between yield spreads at time t and consumption growth at time t+1, $\mathrm{Cov}\left(y^{\$}_{t,60} - y^{\$}_{t,3}, g_{t+1}\right)$, can be written as

$$\left[\frac{D^{\$}_{1,3}}{3} - \frac{D^{\$}_{1,60}}{60}\right] \mathrm{Var}(x_t) + \left[\frac{D^{\$}_{3,3}}{3} - \frac{D^{\$}_{3,60}}{60}\right] \mathrm{Cov}(x_t, x^{\pi}_t), \tag{25}$$

where the EIS ψ, enters the 1st term since

$$\left[\frac{D^{\$}_{1,3}}{3} - \frac{D^{\$}_{1,60}}{60}\right] = \frac{1}{\psi(1-\rho)}\left[\frac{-(1-\rho^3)}{3} + \frac{(1-\rho^{60})}{60}\right].$$

The EIS governs the direct impact on interest rates from changes in growth.

The loadings in front of the variance and covariance terms are both negative, which reflects a flattening or even an inversion of the yield curve in response to higher consumption growth and inflation. That is, short rates increase more than long rates. A rise in x_t, which pushes up g_{t+1}, has 2 effects on yield spreads. First, the yield curve flattens or inverts as short rates are more sensitive to economic growth than long rates. This is a real effect and is governed by the 1st term in expression (25). Second, the yield curve steepens, since positive shocks to growth are estimated to have a negative impact on inflation. This is a nominal effect and is governed by the 2nd term in expression (25). A low EIS implies a high unwillingness to substitute consumption intertemporally and increases the impact of expected growth on interest rates, leading to a sharper flattening or inversion of the yield curve. Hence, a low EIS makes the real effect dominate, which translates into a counterfactual negative relation between yield spreads and future growth. The EIS therefore needs to be high in order for the nominal effect to dominate.

Panel A of Table 13 reports that the estimated value of EIS, 2.51, produces a similar regression coefficient as in the data, while setting the EIS to 0.10 produces a counterfactually negative coefficient. The same table also shows that a high EIS

TABLE 13

Predictability and the EIS

Table 13 reports the effect of changing the elasticity of intertemporal substitution (EIS) when predicting future consumption growth and consumption growth volatility using the nominal yield spread and price-dividend ratio, respectively. The forecast horizon is set to 4 quarters. Reported results refer to population coefficients obtained from simulating 1 sample of 150,000 months. The sample period is 1952:2–2007:4.

EIS	Model β_k	Data β_k
Panel A. Predicting Consumption Growth with the Yield Spread		
$\psi = 2.51$	0.51	0.43
$\psi = 0.10$	−0.76	0.43
Panel B. Predicting Consumption Growth Volatility with the Price-Dividend Ratio		
$\psi = 2.51$	−0.69	−0.53
$\psi = 0.10$	0.14	−0.53

matches the negative relation between price-dividend ratios and macroeconomic volatility, while a low EIS leads to a counterfactual positive slope coefficient. Economically, an EIS above 1 means that the intertemporal substitution effect dominates the wealth effect, which leads the agent to sell risky assets in anticipation of bad times, leading to a drop in asset valuation ratios.

V. Conclusion

Despite the voluminous literature on representative-agent models and their implications for equity and bond prices, less work has been done on modeling the 2 asset classes jointly. This paper evaluates whether the so-called long-run risk framework provides a useful framework for interpreting both equity and bond markets as well as the relation between asset prices and the macroeconomy.

I find that persistent shocks to expected consumption growth together with a negative correlation between U.S. consumption growth and inflation indeed can account for the average level of both equity and bond risk premia, while time variation in macroeconomic volatility can account for evidence of predictability across both markets. The model is able to jointly reproduce the equity premium, the upward-sloping nominal yield curve, and the ability of price-dividend ratios and nominal yield spreads to predict future asset returns, economic growth, and macroeconomic volatility. The model is estimated using a simulation estimator that takes into account time aggregation of consumption growth while utilizing a rich set of moment conditions covering macro variables, equity markets, and bond markets. I also include predictive regressions as moment conditions, which helps identify key parameters such as the EIS.

The model presented here could be extended in several ways. For example, a similar type of model that allows for time-varying market prices of risk (e.g., Le and Singleton (2010)) could potentially generate even stronger predictability of excess returns. The reduced-form approach of modeling inflation in the paper is sufficient for capturing the correlation between economic growth and inflation but is quiet on the underlying mechanisms. I believe further work on endogenizing inflation could yield valuable insights, for example, by allowing monetary policy to play a role (e.g., Gallmeyer et al. (2007)). I leave this for future research.

References

Abel, A. B. "Asset Prices under Habit Formation and Catching Up with the Joneses." *American Economic Review*, 80 (1990), 38–42.

Abel, A. B. "Risk Premia and Term Premia in General Equilibrium." *Journal of Monetary Economics*, 43 (1999), 3–33.

Ang, A., and G. Bekaert. "Stock Return Predictability: Is It There?" *Review of Financial Studies*, 20 (2006), 651–707.

Ang, A.; M. Piazzesi; and M. Wei. "What Does the Yield Curve Tell Us about GDP Growth?" *Journal of Econometrics*, 131 (2006), 359–403.

Backus, D. K.; A. W. Gregory; and S. E. Zin. "Risk Premiums in the Term Structure: Evidence from Artificial Economies." *Journal of Monetary Economics*, 24 (1989), 371–399.

Backus, D. K.; B. R. Routledge; and S. E. Zin. "The Cyclical Component of U.S. Asset Returns." Working Paper, Carnegie Mellon University (2010).

Backus, D. K., and S. E. Zin. "Reverse Engineering the Yield Curve." NBER Working Paper 4676 (1994).

Bansal, R.; R. F. Dittmar; and C. T. Lundblad. "Consumption, Dividends, and the Cross Section of Equity Returns." *Journal of Finance*, 60 (2005), 1639–1672.

Bansal, R.; A. R. Gallant; and G. Tauchen. "Rational Pessimism, Rational Exuberance, and Asset Pricing Models." *Review of Economic Studies*, 74 (2007), 1005–1033.

Bansal, R.; V. Khatchatrian; and A. Yaron. "Interpretable Asset Markets?" *European Economic Review*, 49 (2005), 531–560.

Bansal, R.; D. Kiku; and A. Yaron. "Risks for the Long Run: Estimation and Inference." Working Paper, Duke University (2007).

Bansal, R., and I. Shaliastovich. "A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets." Working Paper, Duke University (2009).

Bansal, R., and A. Yaron. "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles." NBER Working Paper 8059 (2000).

Bansal, R., and A. Yaron. "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles." *Journal of Finance*, 59 (2004), 1481–1509.

Bollerslev, T. "Generalized Autoregressive Conditional Heteroskedasticity." *Journal of Econometrics*, 31 (1986), 307–327.

Brandt, M. W., and K. Q. Wang. "Time-Varying Risk Aversion and Unexpected Inflation." *Journal of Monetary Economics*, 50 (2003), 1457–1498.

Buraschi, A., and A. Jiltsov. "Habit Formation and Macroeconomic Models of the Term Structure of Interest Rates." *Journal of Finance*, 62 (2007), 3009–3063.

Campbell, J. Y. "Bond and Stock Returns in a Simple Exchange Model." *Quarterly Journal of Economics*, 101 (1986), 785–804.

Campbell, J. Y. "Intertemporal Asset Pricing without Consumption Data." *American Economic Review*, 83 (1993), 487–512.

Campbell, J. Y. "Understanding Risk and Return." *Journal of Political Economy*, 104 (1996), 298–345.

Campbell, J. Y. "Asset Prices, Consumption and the Business Cycle." In *Handbook of Macroeconomics*, J. B. Taylor and M. Woodford, eds. Amsterdam: Elsevier Science, North-Holland (1999).

Campbell, J. Y., and J. H. Cochrane. "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior." *Journal of Political Economy*, 107 (1999), 205–251.

Campbell, J. Y.; A. W. Lo; and A. C. MacKinlay. *The Econometrics of Financial Markets.* Princeton, NJ: Princeton University Press (1997).

Campbell, J. Y., and R. J. Shiller. "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors." *Review of Financial Studies*, 1 (1988), 195–228.

Campbell, J. Y., and R. J. Shiller. "Yield Spreads and Interest Rate Movements: A Bird's Eye View." *Review of Economic Studies,* 58 (1991), 495–514.

Campbell, J. Y., and L. M. Viceira. "Who Should Buy Long-Term Bonds?" *American Economic Review*, 91 (2001), 99–127.

Chen, X.; J. Favilukis; and S. C. Ludvigson. "An Estimation of Economic Models with Recursive Preferences." Working Paper, New York University (2008).

Cochrane, J. H., and L. P. Hansen. "Asset Pricing Explorations for Macroeconomics." In *NBER Macroeconomics Annual*, O. J. Blanchard and S. Fischer, eds. Cambridge, MA: MIT Press (1992).

Cochrane, J. H., and M. Piazzesi. "Bond Risk Premia." *American Economic Review*, 95 (2005), 138–160.

Constantinides, G. M. "Habit Formation: A Resolution of the Equity Premium Puzzle." *Journal of Political Economy*, 98 (1990), 519–543.

Constantinides, G. M., and D. Duffie. "Asset Pricing with Heterogeneous Consumers." *Journal of Political Economy*, 104 (1996), 219–240.

Cox, J. C.; J. E. Ingersoll Jr.; and S. A. Ross. "A Re-Examination of Traditional Hypotheses about the Term Structure of Interest Rates." *Journal of Finance*, 36 (1981), 769–799.

Cox, J. C.; J. E. Ingersoll Jr.; and S. A. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*, 53 (1985), 385–408.

Dai, Q., and K. J. Singleton. "Expectation Puzzles, Time-Varying Risk Premia, and Affine Models of the Term Structure." *Journal of Financial Economics,* 63 (2002), 415–441.

Donaldson, J. B.; T. Johnsen; and R. Mehra. "On the Term Structure of Interest Rates." *Journal of Economic Dynamics and Control*, 14 (1990), 571–596.

Duffie, D.; M. Schroder; and C. Skiadas. "A Term Structure Model with Preferences for the Timing of Resolution of Uncertainty." *Economic Theory*, 9 (1997), 3–22.

Duffie, D., and K. J. Singleton. "Simulated Moments Estimation of Markov Models of Asset Prices." *Econometrica*, 61 (1993), 929–952.

Dunn, K. B., and K. J. Singleton. "Modeling the Term Structure of Interest Rates under Non-Separable Utility and Durability of Goods." *Journal of Financial Economics*, 17 (1986), 27–55.

Engle, R. F. "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation." *Econometrica*, 50 (1982), 987–1007.

Epstein, L. G., and S. E. Zin. "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework." *Econometrica*, 57 (1989), 937–968.

Eraker, B. "Affine General Equilibrium Models." *Management Science*, 54 (2008), 2068–2080.

Estrella, A., and G. A. Hardouvelis. "The Term Structure as a Predictor of Real Economic Activity." *Journal of Finance*, 46 (1991), 555–576.

Evans, M. D. D. "Real Rates, Expected Inflation, and Inflation Risk Premia." *Journal of Finance*, 53 (1998), 187–218.

Evans, M. D. D. "Real Risk, Inflation Risk, and the Term Structure." *Economic Journal*, 113 (2003), 345–389.

Fama, E. F., and R. B. Bliss. "The Information in Long-Maturity Forward Rates." *American Economic Review,* 77 (1987), 680–692.

Fama, E. F., and K. R. French. "Dividend Yields and Expected Stock Returns." *Journal of Financial Economics*, 22 (1988), 3–25.

Gallmeyer, M. F.; B. Hollifield; F. J. Palomino; and S. E. Zin. "Arbitrage-Free Bond Pricing with Dynamic Macroeconomic Models." *Federal Reserve Bank of St. Louis Review*, 89 (2007), 305–326.

Hall, R. E. "Intertemporal Substitution in Consumption." *Journal of Political Economy*, 96 (1988), 339–357.

Heaton, J. "The Interaction between Time-Nonseparable Preferences and Time Aggregation." *Econometrica*, 61 (1993), 353–385.

Heaton, J. "An Empirical Investigation of Asset Pricing with Temporally Dependent Preference Specifications." *Econometrica*, 63 (1995), 681–717.

Le, A., and K. J. Singleton. "An Equilibrium Term Structure Model with Recursive Preferences." *American Economic Review*, 100 (2010), 557–561.

Lee, B.-S., and B. F. Ingram. "Simulation Estimation of Time-Series Models." *Journal of Econometrics*, 47 (1991), 197–205.

Lewellen, J. "Predicting Returns with Financial Ratios." *Journal of Financial Economics*, 74 (2004), 209–235.

Newey, W. K., and K. D. West. "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix." *Econometrica*, 55 (1987), 703–708.

Piazzesi, M., and M. Schneider. "Equilibrium Yield Curves." In *NBER Macroeconomics Annual*, M. Woodford, D. Acemoglu, and K. Rogoff, eds. Cambridge, MA: MIT Press (2006).

Restoy, F., and P. Weil. "Approximate Equilibrium Asset Prices." *Review of Finance,* 15 (2011), 1–28.

Stambaugh, R. F. "Predictive Regressions." *Journal of Financial Economics*, 54 (1999), 375–421.

Stock, J. H., and M. W. Watson. "New Indexes of Coincident and Leading Indicators." In *NBER Macroeconomics Annual*, O. J. Blanchard and S. Fischer, eds. Cambridge, MA: MIT Press (1989).

Sundaresan, S. M. "Intertemporally Dependent Preferences and the Volatility of Consumption and Wealth." *Review of Financial Studies*, 2 (1989), 73–89.

Vissing-Jorgensen, A., and O. P. Attanasio. "Stock-Market Participation, Intertemporal Substitution, and Risk Aversion." *American Economic Review*, 93 (2003), 383–391.

Wachter, J. A. "A Consumption-Based Model of the Term Structure of Interest Rates." *Journal of Financial Economics*, 79 (2006), 365–399.

Working, H. "Note on the Correlation of First Differences of Averages in a Random Chain." *Econometrica*, 28 (1960), 916–918.